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HIGH FREQUENCY (HF) AMPLITUDE MODULATION (AM) SIGNAL-TO-INTERFE--ETC(U)
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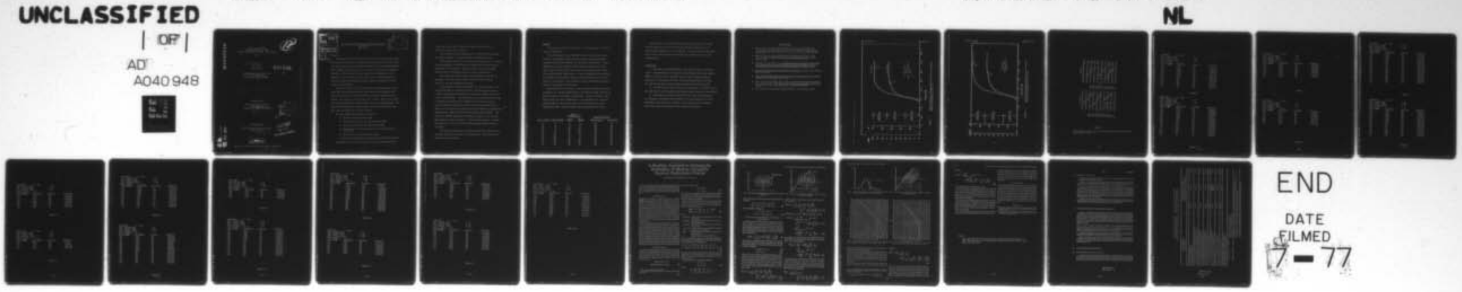

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High Frequency (HF) Amplitude Modulation (AM) SIGNAL-TO-INTERFERENCE RATIO (SIR) STUDY

BY

(10)

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HF AM Signal-to-Interference Ratio (SIR) Study

E.R. Freeman
H.M. Sachs

General:

The objectives of this study are to determine reasonable S/I ratios for high frequency audio modulation (HF AM) Navy Tactical communications, to compare these ratios with current International Frequency Registration Board (IFRB) specifications, and to determine the effects of modifying the SIR's. The approach taken was to develop a mathematical model capable of relating signal, noise, interference and probability of achieving required Signal-to-Noise Ratios (SNR's) and Signal-to-Interference Ratios (SIR's).

The model selected was that developed by Sachs (attachment A-equation 18).[1] The International Radio Consulting Committee (CCIR) model (report 264-2, para-4, attachment B)[2] only considers signal and interference level and does not take noise into account. Sachs' model allows for the consideration of a minimum SNR as well as an SIR threshold. The statistical distribution of both signals are assumed to be log-normal. The model was programmed on a Nova computer.

The data required for input to the model is:

1. Minimum detectable signal (dBm)
2. Standard deviation of the desired signal (dB).
3. Standard deviation of the interference (dB).
4. Correlation coefficient between signal and interference (-1 to +1).
5. Acceptable minimum SIR threshold (dB).
6. Required probability of achieving the minimum SIR threshold.

The output of the model is expected signal level, (dBm) expected

interference level (dBm), expected SIR (dB) and probability of successful communication. (P(R+))

The minimum detectable signla level is a receiver characteristic and readily available. A typical level of -104 dBm was chosen.

The standard deviation of the signal and interference is a function of the propagation path and changes in antenna gain caused by physical fluctuation of the antenna. Studies of various reports 3,4 showed that 8-10 dB is a reasonable range for coastal communications.

The correlation for the signal and interference levels was not available. The CCIR assumes a value of 0.5, but there does not seem to be any data supporting this number.

The acceptable minimum SIR threshold is a direct function of the articulation index or articulation score. The articulation index is a nubmer representing the proportion of unrelated syllables understood by the average listener. The articulation score represents the per cent of English text which would be understood by the average listener. The relationship of these values for AM voice modulation (A^3) interference to an A3 signal was analyzed by the Electromagnetic Compatibility Analysis Center (ECAC)[5] and Rome Air Development Center (RADC)[6]. The results of the ECAC analysis are contained in figures 1 and 2 and the RADC results are shown in figure 3. Based on these studies, a minimum threshold level of 8 to 10 dB for English language messages appears reasonable.

The reguired probability of achieving acceptable communication is specified as 84% for most types of communications. This threshold was utilized for this analysis.

2

Results

The model was exercised for ranges of reasonable values as shown in figures 4 through 20.

The model operates using selected values of receiver minimum detectable signal (MDS), signal and interference standard deviations, signal-to-interference threshold and correlation coefficient. It then steps the mean signal level in 2 dB increments starting at the MDS level. For each increment it steps the mean interference level from 10 to 40dB below the signal level. If any value (R+) of the probability of successful communications equals or exceeds 0.84 it prints out the values of mean signal, mean interference level, mean SIR and the probability of successful communications.

Note the effect of the choice of the correlation coefficient and the convergence on a single expected SIR as the signal strength increases to where the noise level becomes insignificant. Figures 4 through 7 show these effects for what is probably the most optimistic case (standard deviation (σ) of only 8 dB for signal and interference and an SIR threshold of 8dB). Typical results are summarized below in Table 1.

Table I
Typical Cases

Corr. Coeff.	SIR THR-(dB)	σ_S(dB)	σ_I(dB)	Required SIR (dB) Strong Signal	Required SIR (dB) Weak Signal
.9	8	8	8	12	15
.5	8	8	8	16	23
0	8	8	8	20	26
.9	8	10	10	13	16
.5	8	10	10	18	26
0	8	10	10	23	28
.9	10	8	8	14	25
.5	10	8	8	18	25
0	10	8	8	22	25
.9	10	10	10	15	18
.5	10	10	10	20	28
0	10	10	10	25	30

For the cases summarized above the average required SIR for strong desired signals is 18 dB and for minimal detectable signals is 24 dB (at the 0.84 probability of success level). It appears from the available data that the values above are reasonable ranges of values for coastal communications.

Conclusions

1. The model used indicates that a mean SIR of 18 dB is required to assure a 0.84 probability of successful communications with strong signals (SNR's of 15 dB or more). Marginal signals require a higher degree of protection - on the order of 24 dB. The current IFRB standard is 22 dB mean SIR for the type of system under discussion (see attachment C).[7]

2. The CCIR model is only valid for strong signals. The values used by the IFRB assume a 0.5 correlation coefficient, with no basis for this choice.

3. Knowledgable researchers agree that the signal and interference are positively correlated, but the degree of the correlation has not been determined. The results are sensitive to the value of the correlation coefficient used and its actual value should be determined.

Bibliography

1. Sachs, H.M., "A Realistic Approach to Defining the Probability of Meeting Acceptable Receiver Performance Criteria", IEEE Transactions on EMC, Vol. EMC-13, No. 4, Nov., 1971 (corrected version)

2. Report 264-2, C.C.I.R. XIIth Plenary Assembly, New Delhi, 1970, Vol. II, Part 2, Ionospheric Propagation (Study Group 6), I.T.U., Geneva, 1970.

3. Guyader, H. and Olson, I., A Methodology for Determining Performance of High Frequency Communication Circuits Over Ionospheric Paths, Code 2120, Naval Electronics Laboratory Center, 13 April 1973.

4. HF Link 11 Requirements for Gapless Communication, Litton Data Systems, April 30, 1975.

5. Communications/Electronics Receiver Performance Degradation Handbook, ESD-TR-73-104, DoD-ECAC, July, 1973.

6. Duff, W.G., and Stemple, H.L., "Voice Communication Factors Resulting from Interference", 1967 IEEE Electromagnetic Compatibility Symposium Record IEEE 27C80, Washington, D.C. July 18-19-20, 1967.

7. IFRB Technical Standards, Fourth Edition, I.T.U., Geneva, 1968.

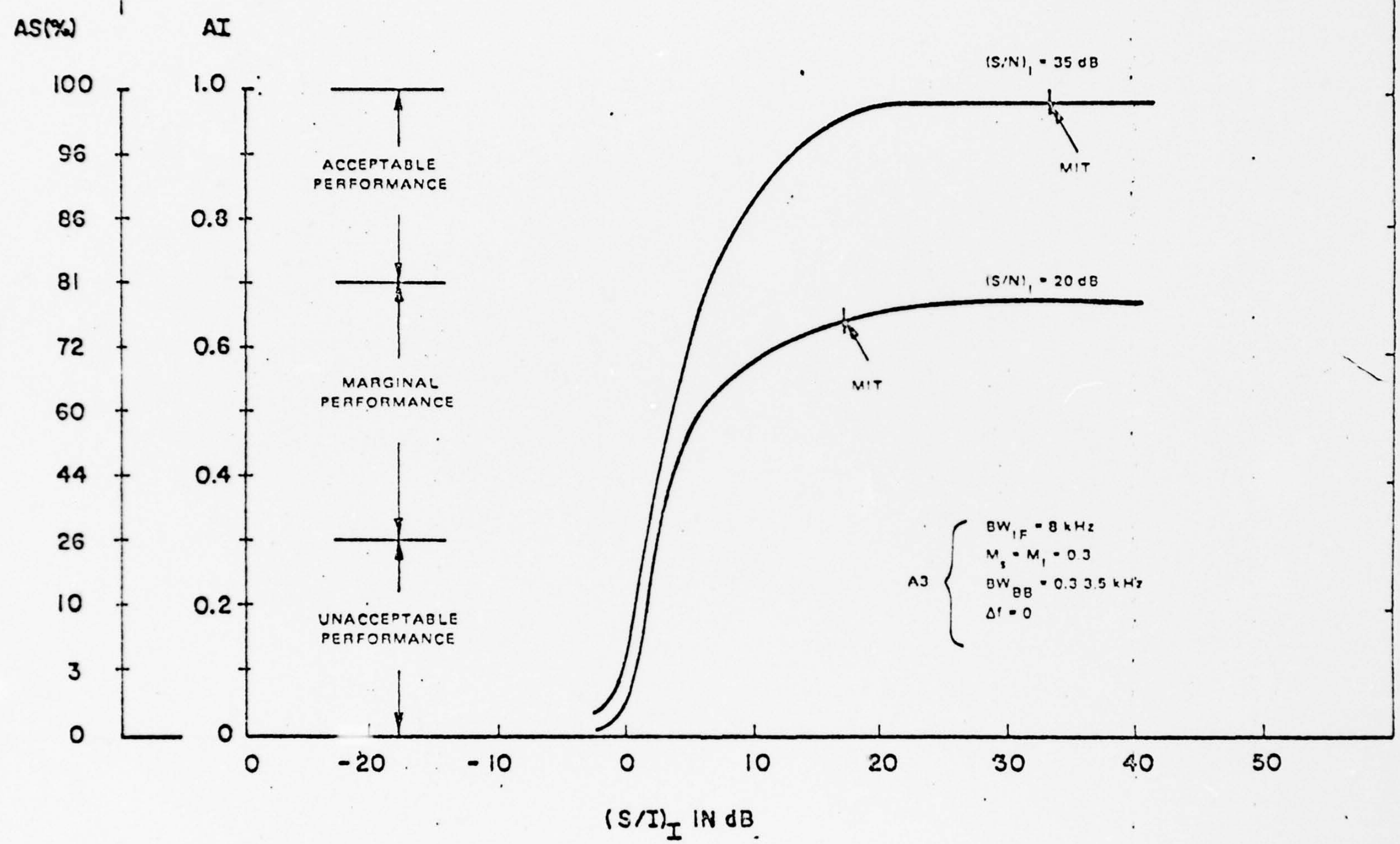


Figure 1 Sample Performance Degradation Curve For A3 Receiver With A3 Interference ($\Delta f = 0$)

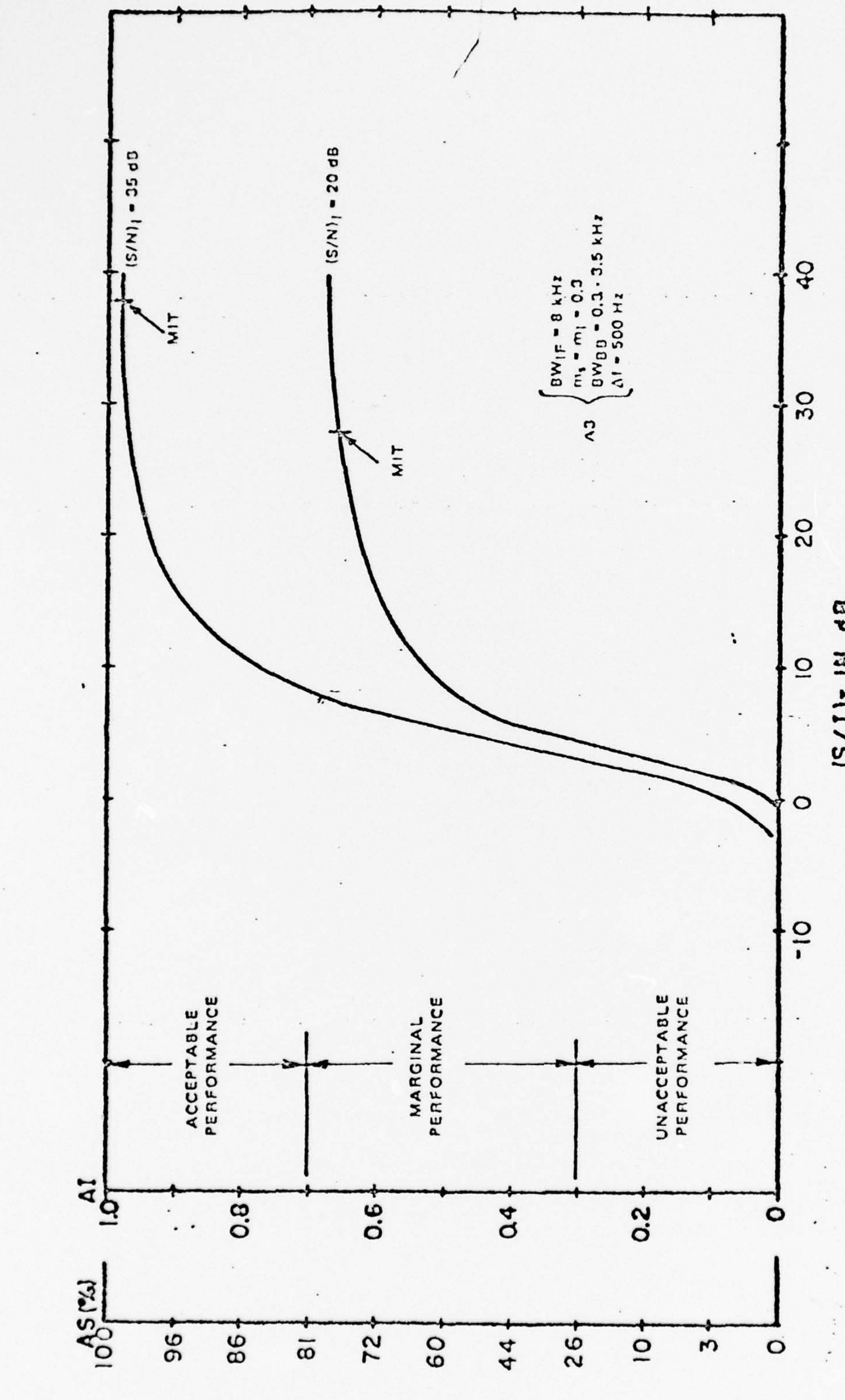
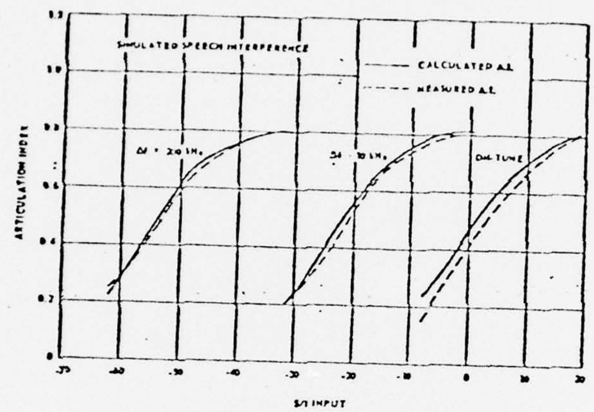
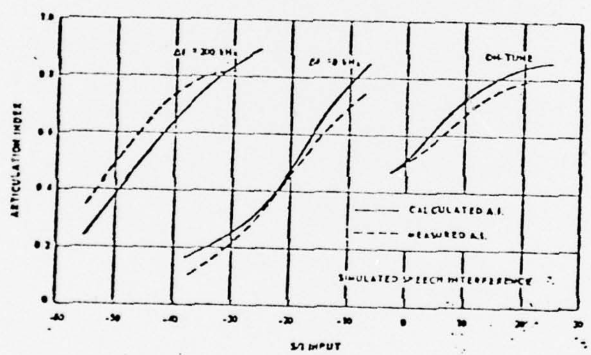


Figure 2 Performance Degradation Curve For A3 Receiver With A3 Interference (Δf = 500 Hz)



Performance Calculated From
Measured Receiver Characteristic



Performance Calculated From
Nominal Receiver Characteristics

Figure 3

Note: Results above based on typical HF AM receiver, bandwidth approximately 8 kHz.

MDS(DBM) ? -104
SIG. SIGMA (DB) ? 8
INT. SIGMA (DB) ? 8
S/I THRESH. (DB) ? 8
CORR. COEFF. ? .9

SIG.(DBM)	INT.(DBM)	S/I(DB)	PROB(R+)
-96	-111	15	.840075
-94	-107	13	.840108
-92	-105	13	.870509
-90	-102	12	.842989
-88	-100	12	.854445
-86	-98	12	.861114
-84	-96	12	.864765
-82	-94	12	.866638
-80	-92	12	.867539
-78	-90	12	.86794
-76	-88	12	.868114
-74	-86	12	.868184

Figure 4

MDS(DBM) ? -104
SIG. SIGMA (DB) ? 8
INT. SIGMA (DB) ? 8
S/I THRESH. (DB) ? 8
CORR. COEFF. ? .5

SIG.(DBM)	INT.(DBM)	S/I(DB)	PROB(R+)
-96	-119	23	.843653
-94	-113	19	.849675
-92	-110	18	.858889
-90	-107	17	.852324
-88	-105	17	.86095
-86	-103	17	.86558
-84	-101	17	.867892
-82	-98	16	.840716
-80	-96	16	.841106
-78	-94	16	.841252
-76	-92	16	.841309
-74	-90	16	.841329
-72	-88	16	.841336
-70	-86	16	.841339
-68	-84	16	.841335

Figure 5

MDS(DBM) ? -104
SIG. SIGMA (DB) ? 8
INT. SIGMA (DB) ? 8
S/I THRESH. (DB) ? 8
CØRR. CØEFF. ? 0

SIG.(DBM)	INT.(DBM)	S/I(DB)	PRØB(R+)
-96	-122	26	.840118
-94	-116	22	.846405
-92	-113	21	.853556
-90	-110	20	.847021
-88	-108	20	.852015
-86	-106	20	.854228
-84	-104	20	.855114

Figure 6

MDS(DBM) ? -104
SIG. SIGMA (DB) ? 8
INT. SIGMA (DB) ? 8
S/I THRESH. (DB) ? 8
CØRR. CØEFF. ? -.9

SIG.(DBM)	INT.(DBM)	S/I(DB)	PRØB(R+)
-95	-119	24	.841519
-93	-117	24	.847085
-91	-115	24	.847528
-89	-113	24	.847542
-87	-111	24	.847543
-85	-109	24	.847537
-83	-107	24	.847539

Figure 7

```
MDS(DBM)          ? -104
SIG. SIGMA (DB)                   ? 10
INT. SIGMA (DB)                   ? 10
S/I THRESH. (DB)                  ? 8
CØRR. CØEFF.      ? .9
```

SIG.(DBM)	INT.(DBM)	S/I(DB)	PRØB(R+)
-93	-109	16	.851441
-91	-106	15	.866289
-89	-103	14	.862627
-87	-101	14	.879252
-85	-98	13	.85055
-83	-96	13	.857574
-81	-94	13	.862044
-79	-92	13	.864765
-77	-90	13	.866363
-75	-88	13	.867257
-73	-86	13	.867737
-71	-84	13	.867992
-69	-82	13	.868114
-67	-80	13	.868177
-65	-78	13	.868201
-63	-76	13	.86821

Figure 8

```
MDS(DBM)          ? -104
SIG. SIGMA (DB)                   ? 10
INT. SIGMA (DB)                   ? 10
S/I THRESH. (DB)                  ? 8
CØRR. CØEFF.      ? .5
```

SIG.(DBM)	INT.(DBM)	S/I(DB)	PRØB(R+)
-94	-120	26	.8405
-92	-114	22	.845643
-90	-110	20	.842023
-88	-108	20	.857952
-86	-105	19	.849441
-84	-103	19	.855793
-82	-101	19	.859642
-80	-99	19	.861864
-78	-96	18	.840256
-76	-94	18	.840821
-74	-92	18	.841106
-72	-90	18	.841236
-70	-88	18	.841298
-68	-86	18	.841321

Figure 9

MDS(DBM) ? -104
SIG. SIGMA (DB) ? 10
INT. SIGMA (DB) ? 10
S/I THRESH. (DB) ? 8
CØRR. CØEFF. ? 0

SIG.(DBM)	INT.(DBM)	S/I(DB)	PRØB(R+)
-93	-121	28	.845799
-91	-116	25	.846578
-89	-113	24	.850388
-87	-110	23	.845504
-85	-108	23	.850459
-83	-106	23	.853132
-81	-104	23	.854484

Figure 10

MDS(DBM) ? -104
SIG. SIGMA (DB) ? 10
INT. SIGMA (DB) ? 10
S/I THRESH. (DB) ? 8
CØRR. CØEFF. ? -.9

SIG.(DBM)	INT.(DBM)	S/I(DB)	PRØB(R+)
-93	-121	28	.84014
-91	-119	28	.84645
-89	-117	28	.847454
-87	-115	28	.847539

Figure 11

```
MDS(DBM)          ? -104
SIG. SIGMA (DB)                    ? 8
INT. SIGMA (DB)                    ? 8
S/I THRESH. (DB)                   ? 10
CØRR. CØEFF.      ? .9
```

SIG.(DBM)	INT.(DBM)	S/I(DB)	PRØB(R+)
-95	-111	16	.848176
-93	-108	15	.856733
-91	-106	15	.881744
-89	-103	14	.849444
-87	-101	14	.858259
-85	-99	14	.863228
-83	-97	14	.865864
-81	-95	14	.867174
-79	-93	14	.867784
-77	-91	14	.868046
-75	-89	14	.868157

Figure 12

```
MDS(DBM)          ? -104
SIG. SIGMA (DB)                    ? 8
INT. SIGMA (DB)                    ? 8
S/I THRESH. (DB)                   ? 10
CØRR. CØEFF.      ? .5
```

SIG.(DBM)	INT.(DBM)	S/I(DB)	PRØB(R+)
-96	-121	25	.843653
-94	-115	21	.849675
-92	-112	20	.858889
-90	-109	19	.852324
-88	-107	19	.86095
-86	-105	19	.86558
-84	-103	19	.867892
-82	-100	18	.840716
-80	-98	18	.841106
-78	-96	18	.841252
-76	-94	18	.841309
-74	-92	18	.841329
-72	-90	18	.841336
-70	-88	18	.841339
-68	-86	18	.841335

Figure 13

```
MDS(DBM)          ? -104
SIG. SIGMA (DB)              ? 8
INT. SIGMA (DB)              ? 8
S/I THRESH. (DB)             ? 10
CØRR. CØEFF.     ? 0
```

SIG.(DBM)	INT.(DBM)	S/I(DB)	PRØB(R+)
-95	-120	25	.841078
-93	-116	23	.844822
-91	-113	22	.842793
-89	-111	22	.849989

Figure 14

```
MDS(DBM)          ? -104
SIG. SIGMA (DB)              ? 10
INT. SIGMA (DB)              ? 10
S/I THRESH. (DB)             ? 10
CØRR. CØEFF.     ? .9
```

SIG.(DBM)	INT.(DBM)	S/I(DB)	PRØB(R+)
-93	-111	18	.851441
-91	-108	17	.866289
-89	-105	16	.862627
-87	-103	16	.879252
-85	-100	15	.85055
-83	-98	15	.857574
-81	-96	15	.862044
-79	-94	15	.864765
-77	-92	15	.866363
-75	-90	15	.867257
-73	-88	15	.867737
-71	-86	15	.867992
-69	-84	15	.868114
-67	-82	15	.868177

Figure 15

MDS(DBM) ? -104
SIG. SIGMA (DB) ? 10
INT. SIGMA (DB) ? 10
S/I THRESH. (DB) ? 10
CORR. COEFF. ? .5

SIG.(DBM)	INT.(DBM)	S/I(DB)	PROB(R+)
-94	-122	28	.8405
-92	-116	24	.845643
-90	-112	22	.842023
-88	-110	22	.857952
-86	-107	21	.849441
-84	-105	21	.855793
-82	-103	21	.859642
-80	-101	21	.861864
-78	-98	20	.840256
-76	-96	20	.840821
-74	-94	20	.841106
-72	-92	20	.841236
-70	-90	20	.841298
-68	-88	20	.841321

Figure 16

MDS(DBM) ? -104
SIG. SIGMA (DB) ? 10
INT. SIGMA (DB) ? 10
S/I THRESH. (DB) ? 10
CORR. COEFF. ? 0

SIG.(DBM)	INT.(DBM)	S/I(DB)	PROB(R+)
-93	-123	30	.845799
-91	-118	27	.846578
-89	-115	26	.850388
-87	-112	25	.845504
-85	-110	25	.850459
-83	-108	25	.853132
-81	-106	25	.854484

Figure 17

```
MDS(DBM)        ? -104
SIG. SIGMA (DB)                 ? 15
INT. SIGMA (DB)                 ? 15
S/I THRESH. (DB)                ? 10
CØRR. CØEFF.    ? .9
SIG.(DBM)       INT.(DBM)       S/I(DB)         PRØB(R+)
-89             -113            24              .845056
-87             -108            21              .846659
-85             -105            20              .853653
-83             -102            19              .851987
-81             -99             18              .841476
-79             -97             18              .852009
-77             -95             18              .860273
-75             -93             18              .86664
```

Figure 18

```
MDS(DBM)        ? -104
SIG. SIGMA (DB)                 ? 15
INT. SIGMA (DB)                 ? 15
S/I THRESH. (DB)                ? 10
CØRR. CØEFF.    ? 0
SIG.(DBM)       INT.(DBM)       S/I(DB)         PRØB(R+)
-87             -125            38              .841663
-85             -121            36              .846582
-83             -117            34              .843722
-81             -114            33              .843351
-79             -112            33              .849216
-77             -109            32              .843211
-75             -107            32              .845806
-73             -105            32              .847506
-71             -103            32              .848586
-69             -101            32              .84925
```

Figure 19

MDS(DBM) ? -104

SIG. SIGMA (DB) ? 8

INT. SIGMA (DB) ? 8

S/I THRESH. (DB) ? 20

CORR. COEFF. ? .9

SIG.(DBM)	INT.(DBM)	S/I(DB)	PROB(R+)
-96	-123	27	.840075
-94	-119	25	.840108
-92	-117	25	.870509
-90	-114	24	.842989
-88	-112	24	.854445
-86	-110	24	.861114
-84	-108	24	.864765
-82	-106	24	.866638
-80	-104	24	.867539
-78	-102	24	.86794
-76	-100	24	.868114
-74	-98	24	.868184

Figure 20

A Realistic Approach to Defining the Probability of Meeting Acceptable Receiver Performance Criteria

HERBERT M. SACHS, SENIOR MEMBER, IEEE

Abstract—A statistical model for establishing the probability of being able to successfully communicate is developed. The model is based on requirements for meeting both a specific signal-to-interference criterion and a specific signal-to-noise criterion, and takes into account the correlation that exists in the variations of desired and undesired path loss.

I. Introduction

MOST statistically oriented interference analysis models generate a receiver interference-to-noise distribution or signal-to-interference distribution and then compute the probability that the interference-to-noise distribution or the signal-to-interference distribution exceeds some threshold criterion in order to identify the expected degree of satisfactory communication receiver performance. When an interference-to-noise distribution is employed, the implicit assumption is made that satisfactory receiver performance is independent of desired signal level. When a signal-to-interference distribution is used, the assumption is usually made that an acceptable signal-to-noise ratio always exists.

Those models that treat both signal-to-noise and signal-to interference effects ignore the possible correlation between the desired signal and the interference. This correlation is far from nonexistent, as evidenced by the path loss variation on long-range HF links where interference signal fades will often occur at the same time as desired signal fades, or by the negative correlation seen by a mobile receiver operating in the region between desired signal and undesired signal base stations. This paper is intended to put all of these statistical concepts into perspective and in particular to show the general rationale that treats signal statistics, interference statistics, and the correlation thereof.

II. Basic Analysis

Consider a receiving system whose performance can be defined on the basis of input signal-to-noise and signal-to-interference threshold criteria. It is desired to specify the probability that such criteria will be met. Under these circumstances, acceptable system performance can be said to occur when

$$\text{signal to noise} \geq R_1$$
$$\text{signal to interference} \geq R_2. \quad (1)$$

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Expressed in units of dB,

$$S - N \geq r_1$$
$$S - I \geq r_2 \quad (2)$$

where R_1 and r_1 designate signal-to-noise ratio thresholds in numeric ratio and dB units, respectively, and R_2 and r_2 designate signal-to-interference ratio thresholds in numeric ratio and dB units, respectively.

Both S and I are functions of the source levels of the respective signals and the gains and losses the signals will incur between the sources and the receiver in question. Thus, for nonmobile systems,

$$S(\mathrm{dB}) = P_d + G_d + G_a - L_d - D(I)$$
$$I(\mathrm{dB}) = P_i + G_i + G_b - L_i - F \quad (3)$$

where

P_d and P_i	desired signal and interference source levels, respectively;
G_d and G_i	desired signal and interference source antenna gains, respectively;
L_d and L_i	desired signal and interference path losses, respectively;
G_a and G_b	receiver antenna gains to desired signal and interference, respectively;
$D(I)$	desensitization of the desired signal by interference; and
F	receiver off-frequency rejection factor.

If the term $D(I)$ is not significant, it has often been shown (for example, see [1], [2]) that, to a first-order approximation, S and I can be represented by

$$S = \bar{P}_d + p_d + \bar{G}_d + g_d + \bar{G}_a + g_a - \bar{L}_d - l_d$$
$$I = \bar{P}_i + p_i + \bar{G}_i + g_i + \bar{G}_b + g_b - \bar{L}_i - l_i - \bar{F} - f \quad (4)$$

where the bars denote the expected values of the parameters of (3), and the corresponding lower case terms represent a sample from a normal distribution describing the variation of that parameter.

Equation (4) can be simplified to the forms

$$S = \bar{S} + e_s$$
$$I = \bar{I} + e_I \quad (5)$$

where

$$\bar{S} = \bar{P}_d + \bar{G}_d + \bar{G}_a - \bar{L}_d$$
$$\bar{I} = \bar{P}_i + \bar{G}_i + \bar{G}_b - \bar{L}_i - F$$

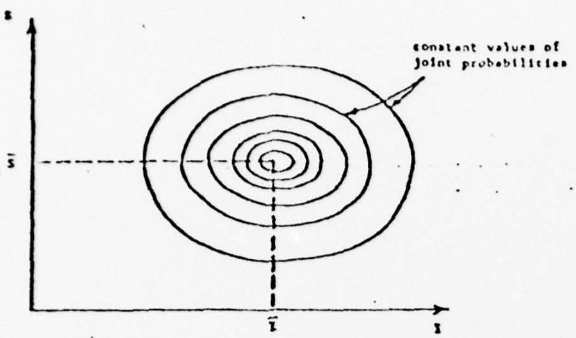

Fig. 1. Joint probability distribution of S and I, assuming statistical independence.

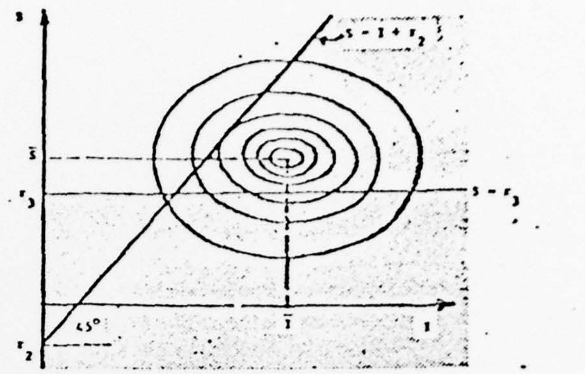

Fig. 2. Conditions for meeting performance criteria, assuming statistical independence.

and e_s and e_I are samples from another normal distribution such that

$$\sigma_{e_s}^2 = \sigma_{p_s}^2 + \sigma_{g_s}^2 + \sigma_{g_a}^2 + \sigma_{l_s}^2$$
$$\sigma_{e_I}^2 = \sigma_{g_I}^2 + \sigma_{g_b}^2 + \sigma_{l_I}^2 + \sigma_f^2. \quad (6)$$

Substituting (5) into (2) gives

$$\bar{S} - N + e_s \geq r_1$$
$$\bar{S} + e_s - (\bar{I} + e_I) \geq r_2. \quad (7)$$

III. Calculations of Probability of Successful Communication

Initially assume that the statistics associated with the desired signal and the statistics associated with the interference signal are uncorrelated. If that is the case, then a plot of equal probability contours of S versus I would look something like that shown in Fig. 1. The joint probability function would be centered at $(\bar{S},\bar{I})$, and would be described by the relationship

$$p(x,y) = \frac{1}{2\pi\sigma_{e_s}\sigma_{e_I}} \exp\left(-\frac{1}{2}\left[\frac{x^2}{\sigma_{e_s}^2} + \frac{y^2}{\sigma_{e_I}^2}\right]\right) \quad (8)$$

where x and y are displacements from $\bar{S}$ and $\bar{I}$, respectively. The plot is actually a three-dimensional one, with the dimensions being S, I, and $p(x,y)$. Now refer again to (2), rewritten as follows:

$$S \geq r_1 + N = r_3 \quad (9)$$
$$S \geq I + r_2 \quad (10)$$

where r_3 is the minimum acceptable signal level. These equations can be superimposed on Fig. 1 to give Fig. 2. Only that portion of the graph left unshaded in Fig. 2 meets the required performance criteria. The total probability of the criteria being met is equal to the volume under the unshaded curve and bounded by the $p(x,y) = 0$ plane and the planes denoted by (9) and (10). This volume can be expressed as

$$P(r_2,r_3) = \frac{1}{2\pi\sigma_{e_s}\sigma_{e_I}} \int_{r_3}^{\infty} \exp\left[-\frac{(S-\bar{S})^2}{2\sigma_{e_s}^2}\right] \cdot \int_{-\infty}^{S-r_2} \exp\left[-\frac{(I-\bar{I})^2}{2\sigma_{e_I}^2}\right] dI\, dS. \quad (11)$$

Equation (11) can be reduced using the substitution $I - \bar{I} = x$ to give

$$P(r_2,r_3) = \frac{1}{2\pi\sigma_{e_s}\sigma_{e_I}} \int_{r_3}^{\infty} \exp\left[-\frac{(S-\bar{S})^2}{2\sigma_{e_s}^2}\right] \cdot \int_{-\infty}^{S-r_2-\bar{I}} \exp\left[-\frac{x^2}{2\sigma_{e_I}^2}\right] dx\, dS. \quad (12)$$

Substitution of $z = (S - \bar{S})/(\sqrt{2}\,\sigma_{e_s})$ in (12) results in

$$P(r_2,r_3) = \frac{1}{\sqrt{2}\pi\sigma_{e_I}} \int_{(r_3-\bar{S})/\sqrt{2}\sigma_{e_s}}^{\infty} \exp(-z^2) \cdot \int_{-\infty}^{\sqrt{2}\sigma_{e_s}z+\bar{S}-r_2-\bar{I}} \exp\left[-\frac{x^2}{2\sigma_{e_I}^2}\right] dx\, dz \quad (13)$$

$$= \frac{1}{\sqrt{\pi}} \int_{(r_3-\bar{S})/\sqrt{2}\sigma_{e_s}}^{\infty} \exp(-z^2) \cdot \Phi\left[\frac{\sqrt{2}\sigma_{e_s}}{\sigma_{e_I}} z + \frac{\bar{S} - r_2 - \bar{I}}{\sigma_{e_I}}\right] dz \quad (14)$$

where $\Phi(\cdot)$ is the cumulative Gaussian probability distribution function. Equation (14) can also be expressed in terms of the error function, giving

$$P(r_2,r_3) = \frac{1}{4}\left[1 - \operatorname{erf}\left(\frac{r_3 - \bar{S}}{\sqrt{2}\sigma_{e_s}}\right) + \frac{2}{\sqrt{\pi}} \cdot \int_{(r_3-\bar{S})/\sqrt{2}\sigma_{e_s}}^{\infty} \exp(-z^2) \cdot \operatorname{erf}\left(\frac{\sigma_{e_s}}{\sigma_{e_I}} z + \frac{\bar{S} - \bar{I} - r_2}{\sqrt{2}\sigma_{e_I}}\right) dz\right]. \quad (15)$$

Equation (14) represents the basic equation for evaluating the probability of successful communications when the signal and interference statistics are uncorrelated. The method of treating this situation has often been to disregard the signal-to-noise requirement and define the preceding probability by the expression

$$P(r_2) = \frac{1}{\sqrt{2\pi}\sqrt{\sigma_{e_s}^2 + \sigma_{e_I}^2}} \cdot \int_{r_2}^{\infty} \exp\left[-\frac{[z - (\bar{S} - \bar{I})]^2}{2(\sigma_{e_s}^2 + \sigma_{e_I}^2)}\right] dz. \quad (16)$$

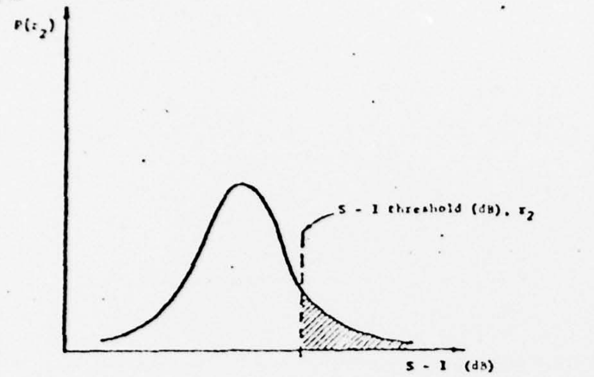

Fig. 3. Customary representation of effects of interference. Shaded area is probability of $S - I$ exceeding threshold.

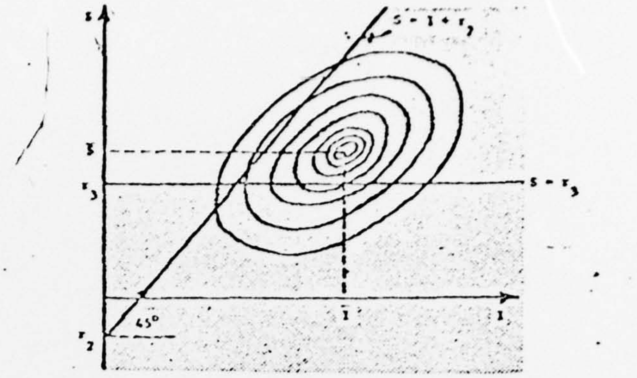

Fig. 4. Conditions for meeting performance criteria, assuming statistical dependence.

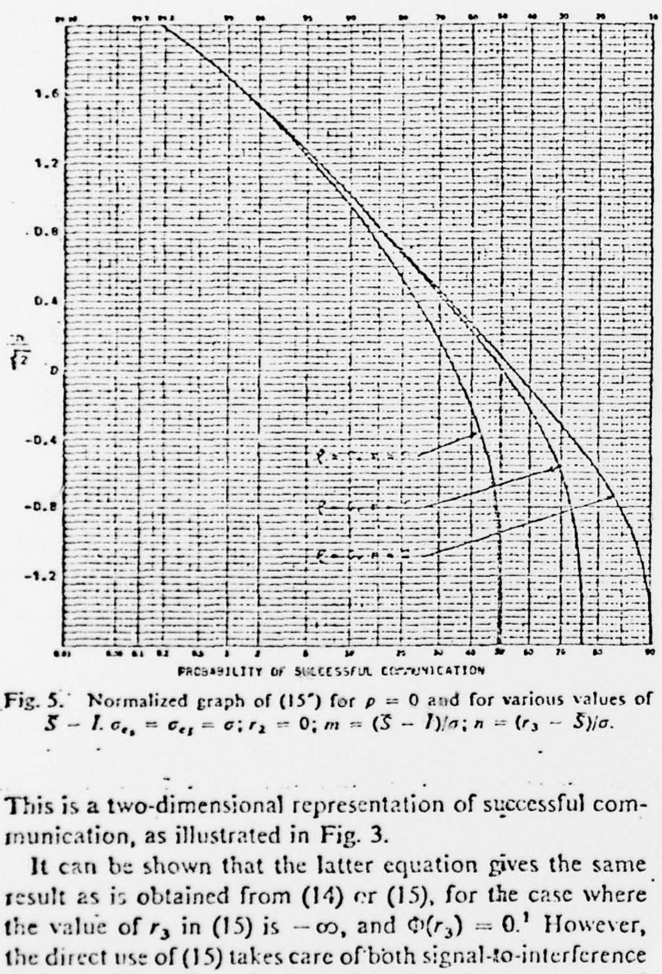

Fig. 5. Normalized graph of (15″) for $\rho = 0$ and for various values of $\bar{S} - \bar{I}$. $\sigma_{e_s} = \sigma_{e_I} = \sigma$; $r_2 = 0$; $m = (\bar{S} - \bar{I})/\sigma$; $n = (r_3 - \bar{S})/\sigma$.

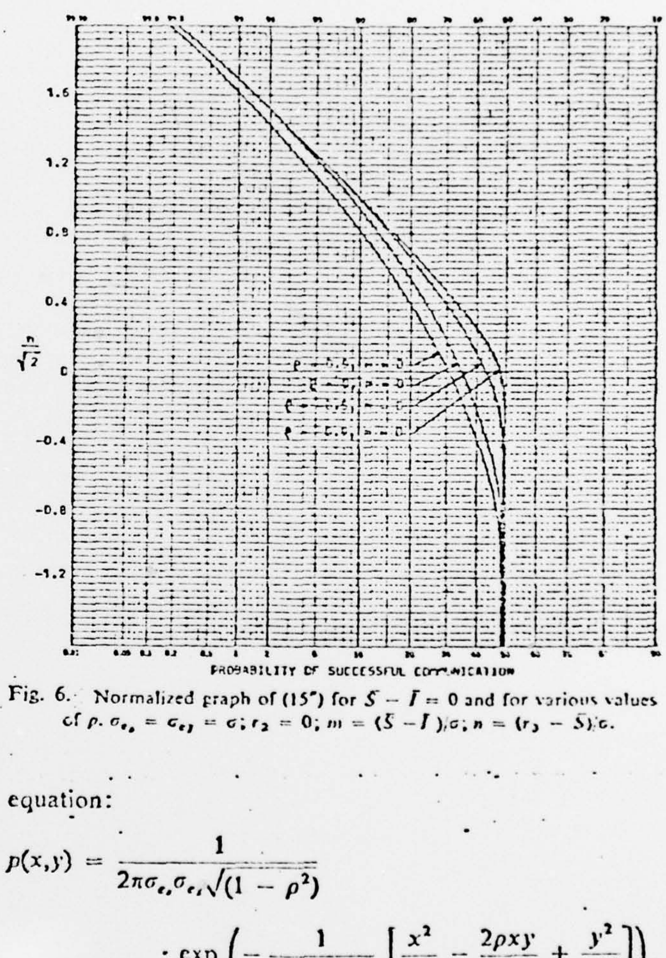

Fig. 6. Normalized graph of (15″) for $\bar{S} - \bar{I} = 0$ and for various values of ρ. $\sigma_{e_s} = \sigma_{e_I} = \sigma$; $r_2 = 0$; $m = (\bar{S} - \bar{I})/\sigma$; $n = (r_3 - \bar{S})/\sigma$.

This is a two-dimensional representation of successful communication, as illustrated in Fig. 3.

It can be shown that the latter equation gives the same result as is obtained from (14) or (15), for the case where the value of r_3 in (15) is $-\infty$, and $\Phi(r_3) = 0$.[1] However, the direct use of (15) takes care of both signal-to-interference and signal-to-noise criteria and is thus the more general expression.

Under the circumstances where correlation between S and I exists, (8) can be expressed as the following equation:

$$p(x,y) = \frac{1}{2\pi\sigma_{e_s}\sigma_{e_I}\sqrt{(1-\rho^2)}} \cdot \exp\left(-\frac{1}{2(1-\rho^2)}\left[\frac{x^2}{\sigma_{e_s}^2} - \frac{2\rho xy}{\sigma_{e_s}\sigma_{e_I}} + \frac{y^2}{\sigma_{e_I}^2}\right]\right) \tag{17}$$

where $\rho = \sigma_{e_se_I}/\sigma_{e_s}\sigma_{e_I}$ is the correlation coefficient between e_s and e_I, and $\sigma_{e_se_I}$ is the standard deviation of the correlated variations. An equivalent to Fig. 2 for the correlated case will appear as in Fig. 4. A similar development to that previously described has been applied to this case as well and has resulted in the relationship given by the following

[1] An interesting integral identity can be established by virtue of this equality. That identity is often found useful when Gaussian assumptions are employed and is

$$-\int_{-\infty}^{\infty} \exp(-z^2)\,\mathrm{erf}\,(az - b)\,dz = \sqrt{\pi}\,\mathrm{erf}\left(\frac{b}{\sqrt{a^2+1}}\right).$$

equation:

$$P(r_2,r_3) = \frac{1}{\sqrt{\pi}} \int_{(r_3-\bar{S})/\sqrt{2}\sigma_S}^{\infty} \exp\{-z^2\} \cdot \Phi\left[\frac{\sqrt{2}(\sigma_S - \rho\sigma_I)z}{\sigma_I\sqrt{1-\rho^2}} + \frac{\bar{S} - r_2 - \bar{I}}{\sigma_I\sqrt{1-\rho^2}}\right] dz. \quad (18)$$

Note that (18) reduces to (14) where $\rho = 0$.

Equation (18) can obviously be plotted in a variety of ways. Fig. 5 indicates a normalized graph of the equation, showing the relationship between $P(r_2,r_3)$ and r_3 for various values of $\bar{S} - \bar{I}$, and for fixed values of $\sigma_{e_s}/\sigma_{e_i}$, r_2, and ρ. It highlights again the sensitivity of $P(r_2,r_3)$ to the receiver signal-to-noise threshold.

Fig. 6 shows the relationship between $P(r_2,r_3)$ and r_3 for various values of correlation coefficient ρ, and for a selected set of values of $\sigma_{e_s}/\sigma_{e_i}$, r_2, and $\bar{S} - \bar{I}$. It gives an indication of the extent to which correlation between signal and interference statistics will influence the probability of successfully communicating.

For mobile systems the variations in L_d and L_i will not only be due to propagation fading effects, but also to variations in path lengths as well, and it is no longer reasonable to assume that l_d and l_i are normally distributed. Thus the joint probability curves must be developed using other than Gaussian relationships. This nonnormal consideration is also true if the desensitization parameter $D(I)$ is not insignificant.

IV. Conclusions

Equation (18) developed in this paper is a more general representation of the probability of successful communication than is customarily used in EMC analysis. It takes into account performance limitations due to both signal-to-interference and signal-to-noise criteria not being exceeded, and thus avoids the strong-signal assumptions of (16). It also considers the degree of correlation that exists between signal and interference variations (when this can be specified) and is therefore not restricted to the usual uncorrelated case.

References

[1] K. G. Heisler and H. J. Hewitt, "Interference notebook," Rome Air Development Center, Griffiss AFB, N.Y., Tech. Rep. RADC-TR-66-1, sec. 1.

[2] A. F. Rashid, "A statistical signal prediction model for communication receivers," *IEEE Trans. Electromagn. Compat.*, vol. EMC-11, May 1969, pp. 90-97.

Note:

This paper has been corrected in accordance with correspondance in IEEE transactions on electromagnetic compatibility, Vol. EMC-14, No. 2 May, 1972, pp 74-78

3.2 *Distribution within an hour*

In Report 266-2 it is suggested that the short-term variations (within a half hour or an hour) follow the Rayleigh distribution.

All the statistical variations considered above refer to the hourly median values. To assess, in a more complete fashion, the possibilities of interference, the quasi-maximum value of the field strength during the course of an hour may well have to be considered.

At any given point of reception, the annual median value of the ratio between the hourly quasi-maximum (10%) value and the hourly median value, varies little from one year to another. This ratio is, however, a function of the distance and the frequency. The study of the median value of this ratio, based on a large number of measurements made during the course of several years, shows that it increases with the frequency and that it decreases when the distance increases. Depending upon the distance and the frequency, this value varies between approximately 6 dB and 3 dB for medium frequencies (band 6) and between 4·5 dB and 2 dB for low frequencies (band 5).

However, for distances where single-hop propagation is no longer possible (above about 2000 km), this ratio no longer obeys an obvious law, but its median value remains in general below 6 dB for medium frequencies and around 2 dB for low frequencies.

4. Formula for estimating the wanted-to-interfering signal ratio R

Consider a particular receiving location, at a distance D_u from the wanted transmitter of power P_u and at a distance D_n from the interfering transmitter of power P_n, and consider an interval of one hour, the mid-point of which corresponds to the local times H_u and H_n of the mid-points of the path of the wanted and unwanted transmissions, then the ratio $R(T)$ in dB between the wanted hourly median signal level and the interfering hourly median signal level, exceeded for a percentage T greater than 50% of the hours of the year when the value R is exceeded, can be calculated for a non-directional receiving antenna from the following formula:

$$R(T) = F_{Hu}(50) - F_{Hn}(50) - \sqrt{\delta_{Hu}{}^2(T) + \delta_{Hn}{}^2(100 - T) + 2\rho\delta_{Hu}(T)\delta_{Hn}(100 - T)} \quad (4)$$

where ρ represents the correlation between the changes in hourly median values for the wanted and interfering signal propagation paths. In the absence of measurements of this factor ρ, it is suggested that it be set equal to 0·5 in using equation (4).

It should be noted that δ_{Hn} and δ_{Hu} always have opposite signs and that the minus sign before the radical in (4) is associated with the practical situation normally encountered, where the time availability T of satisfactory service is greater than 50%.

Strictly speaking, equation (4) is applicable only to the extent that a log-normal distribution describes the data. However, for the distributions encountered in practice, the formula is an adequate approximation. It neglects the rapid variations of both the wanted and interfering signals.

5. Temporal variations of the field strengths

5.1 *Combined influence of the hour and season*

Fig. 6 provides a correction of the hourly median as a function of the hour at the mid-point of the path. But this correction term is itself no more than a yearly median derived from results obtained with different frequencies and at all times of the year. The spread of the correction, shown in Fig. 7 is, therefore, very great. Its value can, however, be rendered

CCIR Method
Attachment B

RAPPORTS DE PROTECTION SIGNAL/BROUILLEUR : Gamme de fréquences de 3500 kHz à 27500 kHz
SIGNAL/INTERFERENCE PROTECTION RATIOS : Frequency range 3500 kc/s - 27500 kc/s
RELACIONES DE PROTECCIÓN SEÑAL/INTERFERENCIA : Gama de frecuencias 3500 kc/s - 27500 kc/s

Type de transmission Type of transmission Tipo de transmisión	Rapports de protection signal/brouilleur Signal/interference protection ratios Relaciones de protección señal/interferencia		
	1	2	
Télégraphie, réception auditive (1) - Telegraphy, aural reception (1) - Telegrafía, recepción auditiva (1)	15(7-14)	15(11-18)	22(13-26)
Télégraphie, météo et presse à réception auditive, système Hell - Telegraphy, meteo, press, aural reception, Hell system - Telegrafía, meteorología, prensa, recepción auditiva, sistema Hell	17(7-16)	21(11-20)	23(13-32)
Télégraphie, réception automatique sans correction d'erreurs - Telegraphy, automatic reception without error correction - Telegrafía, recepción automática sin corrección de errores	26(13-25)	30(17-29)	32(19-31)
Télégraphie, réception automatique avec correction d'erreurs - Telegraphy, automatic reception with error correction - Telegrafía, recepción automática con corrección de errores	14(8-13)	18(12-17)	28(24-39)
Phototélégraphie - Phototelegraphy - Fototelegrafía	29(18-27)	32(22-31)	34(24-33)
Téléphonie, sans raccordement au réseau téléphonique public (1) - Telephony, not for connection to the public network (1) - Telefonía, sin conexión a la red de servicio público (1)	18(13-17)	22(17-21)	24(15-23)
Téléphonie, raccordement au réseau téléphonique public - Telephony, for connection to the public network - Telefonía, con conexión a la red de servicio público	32(24-31)	36(28-35)	38(30-37)
Radiodiffusion - Broadcasting - Radiodifusión	34(26-33)	38(30-37)	40(32-35)
Service mobile aéronautique, télégraphie ou téléphonie - Aeronautical mobile service, telegraphy or telephony - Servicio móvil aeronáutico, telegrafía e telefonía		25	

Renvoi (1) - Note (1) - Nota (1) - Sauf pour le service mobile aéronautique
Except in aeronautical mobile service
Excepto en el servicio móvil aeronáutico

Renvoi (2) - Note (2) - Nota (2) - Dans l'application de la procédure spécifiée à l'article 10 du Règlement des radiocommunications, le Comité considère un rapport de protection calculé inférieur à 23 dB comme dénotant un cas d'incompatibilité apparente.

In the application of the procedure prescribed in Article 10 of the Radio Regulations, a calculated protection ratio of less than 23 dB is taken by the Board to indicate an apparent incompatibility.

En la aplicación del procedimiento prescrito en el Artículo 10 del Reglamento de Radiocomunicaciones, la Junta considera que una relación de protección calculada de menos de 23 dB indica una incompatibilidad aparente.